

Chapter 5. Loopy Logic

Crises in the Foundations of Mathematics

In the early 20th century three major schools of mathematics emerged, each with its own philosophy and a general interest in clarifying the foundations of mathematics. The schools are usually described as Logicism, Intuitionism, and Formalism. For a broader breakdown of how mathematicians interpret what mathematics is “really” about, see the **Wikipedia** article “Philosophy of Mathematics”. There you will encounter the three major trends I just mentioned plus many other nuances: realism (mathematical entities exist independent of the mind); anti-realism (mathematical states have truth value, but do not correspond to a realm of immaterial or non-empirical entities); Platonism (mathematics exists beyond space and time in the world of Platonic forms); Aristotelianism (mathematics arises from real world properties such as symmetry, continuity, and order); empiricism (we learn mathematics from empirical research); monism (the physical world is an extension of the mathematical world); conventionalism (choose axioms for the results they produce); psychologism (mathematical concepts are based on psychological laws); constructivism (proof must be constructible, and no proof by contradiction is allowed); finitism (no infinities or infinite steps); structuralism (mathematical theories describe structures), embodied mind theories (mathematics is a human mental construct, and number arises from counting discrete objects); fictionalism (mathematics is a dispensable body of falsehoods and number is unnecessary to it); social realism (mathematics is a social construct and product of culture); and so on. Obviously some trends are more popular than others, and some mathematicians can fit into more than one “school”. Each of these viewpoints regarding mathematics has its own value and contribution to understanding what mathematics really is, its purpose, and potential. Below I give a few more detailed comments about the three main schools.

Logicism began to develop under Leibniz and became fully articulated under Russell, Whitehead, and Frege, with further refinements added by Wittgenstein, Chwistek, Ramsey, Langford, Carnap, Quine, and others. The agenda is to build the structure of mathematics on the basis of logic and set theory underlying the natural numbers and real numbers. At the basis of a mathematical system are at least two undefined primitive notions plus a list of postulates. On this basis are erected a calculus of propositions, a theory of classes, and relations from which the natural numbers arise, and thence all of mathematics. Unfortunately set theory has been found to contain paradoxes and contradictions that arise from self referral among sets. So a theory involving a hierarchy of types was evolved to avoid these contradictions. A major criticism of the logicist school is that it makes use of mathematical ideas (such as iteration) in formulating its logic, whereas the logic is supposed to be the foundation for the mathematical ideas.

The **Intuitionists** evolved from the ideas of Kronecker, Poincaré, and Brouwer. They started with an intuitive notion of the natural numbers and eventually developed a strict adherence to finite constructive methods. They disallow the law of the excluded middle and maintain that if a proof of a proposition can not be constructed in a finite number of steps, we can not say it is either true or false. Thus that law only holds for finite sets, not infinite sets. Essentially the intuitionists have developed their own version of logic.

The major criticism directed at this school is that its rigorous restriction to constructible proofs renders certain areas of mathematics inaccessible to many mathematicians who are strongly attached to those areas that are still not susceptible to constructivist proofs. At least the intuitionist approach appears not to lead into any contradictions in what it has accomplished.

The **Formalists** begin with Hilbert's study of postulational geometry. Hilbert set out to resolve the set theory paradoxes and address the criticisms of the intuitionists. With von Neumann, Bernays, and Ackermann he worked on developing proofs of consistency by developing rules of the game for working with the symbols so that no contradictions or inconsistencies would arise. Hilbert was able to show consistency for some simple systems, but not for the whole rich system of mathematics. In 1931 Kurt Gödel showed that it is impossible to achieve consistency in a sufficiently rich formal mathematical system such as Hilbert envisioned. Beyond that he also proved that Hilbert's system was incomplete and inevitably contained undecidable problems.

Thus mathematicians today remain unable to resolve the crises in mathematics brought on by the discovery of fundamental paradoxes, inconsistencies, undecidable issues, and incompleteness. Partially this is due to the way in which the mental world of mathematics integrates with the physical world of experiences (as we discussed initially in chapter one.) Another reason for this is that the mental world in which mathematics is explored is by its nature embedded directly in undefined awareness, a state that lends itself to notions of infinity and continuity, whereas the phenomena of the physical world from which mathematicians and scientists model their ideas about logic, numericity, and mathematics does not appear on close inspection to be either infinite or continuous. Nevertheless, there is some evidence that the physical world may be in some way a projection of the mental world and that perhaps is why the precise mental tool of mathematics is so useful in describing physical systems.

Loopy Logic

As a result of the logical crisis in mathematics we end up with a circular quality of what I call loopy logic that lurks deep in the foundations of the way mathematicians organize their mental worlds. Definitions tend to be circular and ultimately based on undefined primitive notions. As a result, logic can take on some of the circularity of the definitions. Furthermore, it may be that the apparent precision of logic does not apply to the physical world and suffers from the same distortion as the abstractly immortal and precise natural numbers when applied to transient physical objects. The set of labels manipulated in the language of logic do not have any necessary connection with real world objects and experiences. This is not to say that the world is without logic. But the validity of a logical argument and its truth value in the real world depend on how well the framer of the argument (the Observer) has mapped his model linguistic argument to actual events that can be verified. And even with a good match important variables may be left out that influence real world outcomes to evolve at variance with mathematical predictions.

Did all of our discussions about real numbers and portions of unity resolve the question of the validity or truth value of Cantor's diagonal? The answer, of course, is a

resounding NO! Questions remain and we must come back to Cantor one more time for further insight, because his study of infinity marks a major watershed in which theoretical mathematicians seriously launched a brave new vanguard from the finite into the infinite. This step took the mathematicians into another dimension of reality. Seeing clearly how this happened is useful for understanding where mathematics is today in terms of challenges and opportunities.

Given localized information about a non-local object that is too weakly defined, we are left trying to guess a quantum particle's state without collapsing its wave function. That does not work. It is hard to count something that is undefined. An entrepreneur once said to the early Coca Cola company when it was selling its drink at soda fountains, "Put it in bottles!" Precisely define your undefined product. Cantor appears to give us bottles (a_{ij} 's and a_{kk} 's), but on closer inspection, rather like the mysterious formula for Coca Cola, we still do not know precisely what is in them. In Cantor's prior explorations of infinity with \mathbb{N} , \mathbb{Z} , and \mathbb{Q} , we always know exactly what numbers are where, and he counts them in an orderly fashion using the natural numbers with no intermediary operation. Not so with the real numbers \mathbb{R} .

In his non-denumerability proof by "contradiction" Cantor **begins** with the real numbers **assumed** already to have been "counted" out into a complete list mapped to the natural numbers. Then he employs an operation that is now called the **axiom of choice** (also called Zermelo's postulate) on his list of real numbers.

The Axiom of Choice: If a set S is divided into a collection of mutually disjoint nonempty subsets A, B, C, \dots , there exists at least one set R which has as its elements exactly one element from each of the subsets A, B, C, \dots . (Eves and Newsom, p. 329)

In Cantor's case S is a "complete" infinite list (E_0 to E_n) of binary sequences of symbols that represent the "real numbers". Actually the sequences of binary symbols in the list represent nonempty subsets of the list: A, B, C , and so on. Each subset (item on his list) consists of an infinite sequence of binary symbols. Cantor uses the axiom of choice to create a special sequence R that passes diagonally through the list by selecting as the special sequence's elements exactly one symbol at position a_{kk} from each row in the array of the subset sequences A, B, C, \dots in the list S , thereby in one shot exercising the choice axiom over the entire infinite list S to generate sequence R . Then he performs another choice operation on his subset R such that he simultaneously flips each element (i.e., the unique element in each minimal subset) of R into its binary complement to form a new sequence E_u .

In this manner Cantor has introduced a new type of operation that was not necessary or used in all of his previous proofs in which he simply mapped each member of the set he was studying into 1-to-1 correspondence with the natural numbers and counted them off in sequence. In the case of his real number list he has shifted from a well-accepted infinite one-by-one counting procedure with the natural numbers to an infinite transformation operation on an infinite set. This bothered a good many of the mathematicians in his day and was considered very controversial. It was not merely a

meta-linguistic issue, it brought to awareness a meta-operation that may have been used before by mathematicians but was not so clearly recognized as such and took mathematics into an entirely new realm of playing with transfinite sets that are by their very nature non-computable by the ordinary rules of arithmetic. An infinite counting procedure is already non-computable and exists only by virtue of a logical belief about a potentially endless iteration. The axiom of choice took mathematicians to a whole new level of belief -- doing operations on infinite sets, and some mathematicians were not ready to jump in and accept that, because the "construction" could not actually be constructed except by a leap of imagination that somehow did not seem logical in the "real" world.

In the array of rationals Cantor constructed each item in each row is a separate precisely defined number that is really just a unique object in the set of "rationals" and need not even be considered a number. Due to the nature of Cantor's listing of real numbers with a lower level language (as we discussed in chapter 1), **we do not even have to use numbers** in the list in order to examine Cantor's diagonal method. However, the major difference is that we do not treat the individual symbols in a row as separate objects when each infinitely long row sequence is taken as a unique element in the overall list of sequences. However, to construct a unique new sequence for the list, Cantor shifts viewpoint and changes one element within each infinite subset row sequence.

Suppose we just use sequences of two arbitrary symbols, **m** and **w**. This approach avoids the awkward problem we encountered with our numerical notation system for representing arbitrarily sized portions of unity and makes it clear that **the sequences in the list are not really numbers**. What we then have is an exercise in **combinatorics**. Finite combinatorics are simple. Let's take a look. Here is a finite list of 3 sequences, each with 3 elements. **For a diagonal to pass through each sequence row on the list, the array must be square.**

E0 = mmm

E1 = mmw

E2 = mw~~m~~

Eu = ww~~w~~

The flipped diagonal ~~www~~ that we derive from mmm is clearly **not** on the list, but it is also clear that the list as given is not **complete**, because the complete finite list has $2^3 = 8$ sequences, and the diagonal is only a maximum of 3 items long (the chosen length for each sequence), so 5 possible combinations of sequence are missing from the list. In fact it is clear from this example that **NO finite list that can be diagonalized will be complete if it is square**, because it is impossible to list out all the finite combinations of a binary system in a **square** array of any size. Even the degenerate array with one symbol **m** has a flipped "diagonal" **w** that is not on the "list". The complete list will always be longer than any possible diagonal. So right away we see that Cantor's use of a "diagonal" over a supposedly "complete" infinite list of infinitely long sequences begins to look very fishy. It simply does not "square up", and we know that for all finite lists

the flipped diagonal of the list **will not be on the finite list as given, but will also be a member of the complete combinatorial list.** Here we have found the source of the problem with Cantor's construction of a "contradiction". Cantor's "diagonal" procedure itself **by definition** is unable to pass through the complete set of sequences, so the complete combinatorial set listed out will always be larger than any diagonal that passes down the list. And the list is already mapped to the natural numbers. Therefore, we capture the combinatorial essence of the binary sequences **not by listing them all**, but by stating that there are 2^n sequences in a complete list of any size, where the natural number n represents the length of each sequence in a given list, and each binary sequence necessarily has its combinatorial complement in the complete list. Then we know for certain that any flipped diagonal will not be on the list, because it will be different from any that we have listed, but it still will be a member of the complete list. This is not a contradiction showing that there are more "real numbers" than natural numbers, since Cantor has already counted his list with the natural numbers. It is how his model is defined and structured. It just looks weird because it seems as if an array that is infinite in both rows and columns should somehow be "square". However, any complete combinatorial list of sequences labeled with natural numbers however big always stays "ahead" of its diagonal and the diagonal can never catch up to the complete list. The List is bigger than the Diagonal. $L > D$. In fact the diagonal falls further and further behind the length of the list as the length of each row sequence grows in size.

E0 = mwmmwmww...
 E1 = wwwwwww...
 E2 = wmmwwmmw...
 E3 = mmmmmmm...
 E4 = mwmwmmm...
 E5 = wwwwmww...
 E6 = mmwwmmww...
 E7 - wwmmwww...

 Eu = wmwmmwmm...

Do you see how deceptive the partial array is when it is drawn as an 8x8 square in this example with lots of dot-dot-dots and a nice diagonal from one corner to the far opposite corner of the partially written list? We saw in our earlier discussion how we can organize our infinite list of infinite sequences so that the sequence of the diagonal of the array is the same as the sequence of the first row on the list. We can use any simple algorithm to define the sequence of the first row of symbols and the diagonal. When we flip the diagonal we definitely get the complement of the first number on the list, which we already have defined to be part of the complete list by our complement rule. This makes the flipped diagonal become the "limit" of the list at the "end" of the list. It is not in the list, but it completes the list with sequences never captured within the diagonalized portion of the complete list according to the design of the game.

With his diagonalizing over an infinite list of infinite sequences Cantor has now shifted our attention from a list of sequences to a meta-list, a list of lists. As we repeat the diagonalizing, at each such operation we get a new list with 1 additional item that forms its limit and the link to the next list. If we put the "new sequence" at the head of the original list, and then diagonalize again, we get a new flipped number that is not on the list and thus a new list with one more sequence on it. So now we can have an endless list of lists (L_u) that exactly corresponds to the sequence of natural numbers. $L_1 = 1$, $L_2 = 2$, $L_3 = 3$, and so on for as far as you like to go. This goes back to Cantor's original counting technique of going one by one, step by step. Each unique list L_n in the meta-list L_u is a member and a subset of L_u . Each subsequent list just adds one more sequence to the list of sequences. Viewed thusly the list of lists never raises the level of "infinity" to any higher transfinite value than the cardinality of the natural numbers. In fact, none of these lists is complete, and none of them actually represent "real numbers". They are just sequences of various combinations of meaningless binary symbols that are put into a list and labeled with the natural numbers.

By our precise mathematical definition of the array **the number of combinations of 2^n unique sequences of binary elements containing n elements, n being any natural number used to define the length of each row sequence in the array, is always greater than the number n of elements that make up any single row sequence, all sequences in the list of combinations being equal in length. ($2^n > n$)** If the sequences are all 30 unique binary elements long, the complete list of possible sequences will be 1,073,741,824 sequences long, a number which is obviously larger than 30, which is the length of each individual sequence. The expressions 2^∞ and ∞ are **not numbers** and are therefore not computable mathematically, but the ratio $2^n/n$ approaches infinity as n approaches infinity. We also can generalize further to say that $b^n > n$, where b is any natural number base. So Cantor's "construction" of a number that is not in a list of all real numbers is logically meaningless and not a contradiction. If asked for an evaluation of the transfinite relation of two countable infinities (b^∞ / ∞), I would give it the value 1, since both represent the "cardinality" of the natural numbers. $N/N=1$.

Modern mathematics as well as physics and other scientific disciplines are replete with such sleights of hand as we have seen in our discussion of Cantor's work with infinite sets. We must be careful when applying mathematical approaches to the study of physical systems, because the mathematical model must computationally fit the physical system it describes. The most common problems in these disciplines arise from the following practices. There may be others, and I will discuss many of these scientific tricks at various points in the essays that follow.

1. "Renormalization" often is used to reset equations arbitrarily when they get into trouble with infinities that appear in the mathematical description of phenomena. (e.g., at black hole event horizons.)
2. Unannounced switching between mathematical systems, models, or levels of discourse (e.g., between metalanguage and lower level language) in different parts of a proof, hypothesis, or experimental procedure produces an illusion of proof but with logical

problems. (Example: Cantor's unannounced switch in his demonstration of "uncountable" real numbers from using finite operations to using transfinite operations.)

3. Equations in complex numbers often are used for convenience of calculating wave functions, but then the imaginary results are thrown away, using only the part of the model that suits what the scientist wants to describe and ignoring the rest, thus spoiling the elegance and completeness of the model. Feynman specifically gives an example of an imaginary result that may be interpreted in physics (**The Feynman Lectures on Physics**, vol. 2, sect. 24-3, "The cutoff frequency"), warning gently about the practice of discarding uninspected imaginary results, even though he often uses the technique without applying deeper inspection.

4. Obscuration of the observer's role as a fundamental component in science is done to produce the illusion of "objectivity" and sometimes also to hide from the student or reader the scientist's personal biases. An example is Newton's $F = ma$. A scientist must interpose himself into an experiment in order to measure a force.

5. In certain "proofs" what I call "dummy" numbers (with no definable precise value) are used as if they actually had precise values. I do not here refer to variables like x and y . An example is Cantor's use of his "precise" a_{ij} and a_{kk} labels in his real number analysis. We have seen that by defining the "dummy" numbers in some way we can reveal properties of a system that may otherwise go unnoticed.

6. Deliberately leaving out components of a mathematical description can obfuscate important relationships that the scientist prefers not to deal with, whether for personal, political, or other reasons. Examples: dropping the constant *alpha* from the "definition" of the Planck mass; leaving out the back propagation of wavelet "bubbles" in the Huygens model of how light wave wave fronts propagate in space.

7. Insistence on a certain paradigm or experimental procedure (that is basically an argument from authority) for reasons of politics, political correctness, funding, etc. Example: Only funding research and publishing papers in peer-reviewed journals on the high price tag hot fusion approach to fusion energy with extreme prejudice toward the so-called "low energy nuclear reactions" (LENR) and other related researches even though the low energy people having spent only millions have managed to generate a few excess calories here and there although not yet of any commercial value, while the hot fusion people also have never produced any usable excess calories after much larger infusions of cash in the 10's of billions. According to quantum mechanics a certain low level of fusion should naturally occur, so why all the political bias toward low end research when low energy reactors would seem much more practical than a miniature star in a box, however flashy that might seem?

8. Use of overly complicated mathematical descriptions so that the general public is excluded from engaging more deeply in the discourse and/or even a simple appreciation of the science. Elegance means beauty and simplicity in mathematical descriptions of nature and expresses the generative power of natural laws. Perhaps this is "just" a belief,

but definitely it is an essential component of any attempt to "explain" the world intelligibly, especially to a wider audience. Examples: string theories, general relativity, many aspects of quantum mechanics.

9. Promotion of mysterious and invisible theoretical physical properties such as gravitons, inflatons, Higgs particles, dark energy, and dark matter as virtually established paradigms before hard evidence is obtained and verified. Such proposals may be considered only hypotheses or predictions until they are confirmed experimentally or disproved by better theories and results.

Cantor's little diagonal game has played an important role in the history of mathematics and is certainly an ingenious device. The countability or un-countability of reals is related to the Continuum Hypothesis, a question that bothered mathematicians for centuries, especially since the development of the calculus of continuous functions by Newton and Leibniz. People generally believed that men like Cauchy with his epsilons and deltas and Dedekind with his complicated ideas about "least upper bounds" and "cuts" put the question to rest. Gödel later showed that, in spite of these clever ideas, whenever you create the postulates for a set of any kind, if you want it to be continuous, whatever you use for your Continuum Postulate is equivalent to the Parallel Postulate in Euclidean Geometry -- and is thus an optional viewpoint. Other viewpoints may be just as consistent. As I showed in chapter 1, Mathis derives the principle of differentiation from basic number theory without recourse to the usual complicated theories of limits or infinitesimals.

For over two thousand years everybody who learned Western geometry believed that Euclid's postulates are just the way it is, a nice ideal model of the real world. Many suspected that the Parallel Postulate was not a postulate, but no one could ever prove it as a theorem. Finally during the 19th century creative thinkers like Gauss, Bolyai, Lobachevsky, Riemann, and Poincaré, began shifting in another direction. Instead of trying to prove the parallel postulate, they discovered that you could safely **change** the parallel postulate and the geometry still worked! Different postulates about "straight" lines produced different kinds of geometry with applications to different environments. So the Parallel Postulate wasn't right or wrong after all -- it was simply an arbitrary viewpoint. It is one way among many that an Observer may look at a system of geometry, which, at its basis, is a system of logic not necessarily even connected to points, lines, and shapes. We can treat these as purely abstract symbols.

Eventually not only geometry, but algebra and every other branch of mathematics, became "liberated". Mathematicians discovered that there are any number of ways to design algebras, geometries, logical systems, and so on. And they are all connected. You can interpret algebra as geometry or as a system of pure logic. Most modern mathematicians have realized that there is no "right" way to do math. Different mathematical systems may have very different interpretations and applications. Not only is the Parallel Postulate arbitrary, every other postulate is arbitrary. As long as you are reasonably consistent you can create any system you like and play with it and see where it leads you. If it is fun to explore, others will come to join you and play with you.

Mathematics has become a field of all possibilities open for exploration, just like consciousness in the field of undefined awareness. Set up some definitions and start exploring. This means that the science of mathematics has already become Observer Mathematics.

Such a radical realization is already fairly well established among contemporary mathematicians even though they don't come out and call their discipline OM. Accepting OM is not too difficult a leap of imagination for them because math is inherently an abstract mental exercise and not an object in the "real" world. We can call it an OM meditation. Thus, playing with transfinite cardinals and ordinals is fine, as long as the mathematician realizes he is playing with the most abstract areas of consciousness and its underlying undefined awareness, but not the expressed physical world that probably ranges over no more than about 100 or more degrees of magnitude and requires mathematical computability. Interesting ideas have come out of the New Age scientists working on a science of consciousness. For example, Paul Corazza, a professor of mathematics at Maharishi University of Management has developed an Axiom of Wholeness that leads to a "top-down" description of set theory. Beginning with a transcendental field or quality of Wholeness, Corazza derives transfinite and then finite sets in a downward hierarchy. Whether his transfinite world ever really comes down for a landing in the physical world is an interesting question.

However, mathematics is also the primary tool that scientists choose to use for describing the "real" world. Most physicists, and other scientists who study the physical world, are still pretty well stuck in the idea that the "real" world is really "real". There is a certain way that things are and behave, and "that's the way it is" -- even though physicists often use this or that different mathematical system to model whatever they are studying. How do they justify the arbitrary nature of this mental model or that mental model? We do science to study "how it is" and to describe that "how-ness" to each other. If you recall our discussion of belief systems in Chapter 2, "this is how it is" corresponds to a Type 1 belief system governed by a belief that something **is** just the way it is, and all we can do is perhaps describe it and definitely abide by it, but never get to the big WHY.

In Cantor's "real number" proof he uses a technique that recalls the classic paradox of the Cretan liar who warns that all Cretans are liars. Is he telling the truth or not? Cantor says his list is complete. Is it or is it not? As the situation is set up you are stuck with an apparent contradiction, because he produces a "number" that appears not to be on the list without ever actually showing you his complete list. A modern version of this logical problem is, "This statement is false." Whenever you create a liar's paradox, you make a negative mental feedback loop. The self-referral forms a destructive interference pattern that causes the logical system to crash. The whole thing dissolves. Another example is the problem of the barber who only and always shaves all men in his town who do not shave themselves. Does he shave himself? If he does, by his rule he must not shave himself. If he does not, then he has obligated himself to shave himself rather than let another barber do the job -- except that his own rule prohibits shaving someone who shaves himself. Notice how the observer-participant is ultimately responsible for the problems he creates for himself by the way he sets up the rules of the game. The

problem in logic occurs when we consider sets that self-refer, -- that are members of themselves. Bertrand Russell pointed out this logical problem at the foundation of logic. If N is the set of all sets that are not members of themselves, and X is *any* set, then if we let X become N , we get that N is a member of itself and N is not a member of itself, which of course is a logical contradiction.

Such a contradiction does not mean that logic is dead, it means we have been operating under a transparent belief that ignores the loop structure of self-referring beliefs. Setting up a straw dog and then generating a contradiction to knock it down is a common proof technique in mathematics. The adventure here is that such methods might at any point call the whole logical system into question, or they might lead to new breakthroughs and insights into the core beliefs that underlie mathematics.

In a binary (bivalent) logical system we usually imagine that things can be either true or they must be false. With the law of excluded middle either a proposition is true or its negation is true. Actually, such clarity is often not the case in the "real" world. Mathematicians have evolved "fuzzy" logic and other types of multiple-valued logic to model such conditions. In quantum mechanics we actually have at least four possibilities (a tetra-lemma) to deal with. Any proposition in a binary logical system may be True (it always gives true results), or it may be False (it always gives false results), or it may be neither True nor False, or both True and False. To clarify what the last two cases are like, consider the proposition, "The weather is hot." The description of "hot" is relative and different people may agree or disagree with that judgment. Here is a self-referring statement: "This statement is False." Such a proposition generates a negative feedback loop of destructive interference and becomes undefined and unreal in terms of a two-value logic. It is grammatically correct and in plain English, but we cannot find its meaning. The belief system short circuits, and we are left with a sentence in which the words make no sense unless we redefine the words. It is a kind of nonsense. Examples that get into logical trouble often have such a quality of self referral. English phrases also can have multiple interpretations. If the sentence is, "This super smart student at MIT is a jerk," we may consider it an odd contradiction. The student may be very intelligent, but socially obnoxious. Or the student may be working his way through college as a soda fountain "jerk" but is both intelligent and socially gracious. The sentence can be true in one sense and false in another sense all at the same time if he is a gracious soda jerk. Here is another example.

- * To many people George was known as rich.
- * To many people George was known as "Rich".

Take the infinite summation: $S = 1 + 1 - 1 + 1 - 1 + \dots$. The total may equal 0, 1, or 0.5 depending on how we parse the addition.

- * $S = (1 - 1) + (1 - 1) + (1 - 1) \dots = 0$
- * $S = 1 - (1 - 1) - (1 - 1) - (1 - 1) \dots = 1$
- * $S = 1 - (1 + 1 - 1 + 1 - 1 + 1 - 1 + \dots)$. $S = 1 - S$. $2S = 1$. $S = 1/2$.

Here we have three reasonable answers for S, the last one proposed by Leibniz. The problem lies in the infinite summation and an ambiguity in the notation that resembles natural language where words commonly have multiple meanings.

Now let us consider another proposition that involves a meta-belief that on the surface resembles the Cretan Liar paradox.

* "You experience what you believe, even if you don't believe it,"

This proposition generates a positive feedback loop of constructive interference. If we probe its meaning, we discover something that is undefined but real. This is a statement -- i.e., an assertion of a belief -- that describes a real experience. It is not just a nonsensical sentence, even though it seems to short-circuit your own belief system at first glance. Closer inspection reveals that the sentence links beliefs to experiences. If you do not believe that beliefs and experiences are linked, that is a different belief. You may indeed experience that there is no link between beliefs and experiences. Events may sometimes, often, or even always seem to happen to you differently from how you believe they ought to happen. That may lead to a lot of complaints and dissatisfaction in life or perhaps a strong sense of resignation. It is very easy for a person who very strongly holds such a belief to be unable to recognize that in fact she is experiencing nothing more than her strongly held belief that events happen differently to her from how she thinks they ought to happen.

Such a belief about reality can be extremely disempowering, but at the same time so transparent and invisible that the person is quite unaware of the extent to which such a belief hobbles the ability to perform successfully in life. On the other hand, recognition of this kind of loopy logic can bring about a powerfully positive liberation of creative energy in a person's life. Such a recognition reveals that, by shifting viewpoint with regard to beliefs, a person may radically change the way he interacts with his world experientially. Perhaps by changing a belief one may change the experience that it engenders. In any case, if you do not believe that you experience what you believe, and are being honest, then you indeed are experiencing just what you believe -- that your experiences do not match up with your beliefs. Palmer's Proposition is a wake-up call to remind us that we sometimes severely limit our potential by the beliefs we hold without even realizing it.

The trick to this meta-belief is that denial of the meta-belief simply reinforces the core paradigm about beliefs presented in the proposition: that belief precedes experience and thus you experience whatever you believe even if you do not believe it, because **not believing** is also a form of **believing**. The circularity here does not destroy the logic, it reinforces it!!

A negative paradox such as "This statement is a lie" is like falling into a logical black hole from which the mind cannot logically escape. The only way out is to outlaw such statements. A positive paradox creates a logical white hole -- a logical universe of infinite real possibilities emerges from it.

The first time I became aware of the existence of such propositions was when I saw a similar statement by Harry Palmer. Here's what he wrote.

* "You experience what you believe, unless you believe you won't, in which case you don't, which means you did."

(**ReSurfacing**, p. 104).

Do you see how he built a feedback loop with a kind of logical constructive interference? That self-referring "interference" generates a real-life experience of stability that we can rely on, just like the quanta of whole number waves for electron orbits makes them dynamically stable for forming the atoms and molecules of our physical world.

Palmer says in his Proposition that, if you have a belief, that belief generates a corresponding experience. This idea is a more general restatement of Maharishi's principle in the Science of Creative Intelligence that for every mental state there is a corresponding physical state. Palmer's restatement includes the notion that the experiences generated by beliefs (beliefs being Maharishi's "mental states") may occur in mental space or in any other nonphysical dimension as well as the physical dimension. Believing you won't experience what you believe is a meta-belief about your own beliefs. It also suggests that you can trace back from your experiences to find out things you really believe strongly enough to influence your experiences but may have forgotten or ignored. It further suggests that by truly changing your beliefs (not just pretending to) you can change your experiences. These ideas are positive and empowering. They also connect to physics. If you really believe that your beliefs cannot influence your experiences, then it does not matter what you believe, and that tends to greatly lower the incentive to hold positive beliefs and act on them. If you believe your beliefs influence the world, then you can change the world you live in by simply changing your beliefs with certainty and then truly living by them.

Palmer's Proposition relates to my discussion of the lens/mirror that reflects between mental and physical states. It defines the connection between Mental Space and World Space. If you direct an opinion onto Palmer's Proposition, you create a self-referral loop. Any opinion you have about Palmer's Proposition is a belief that you hold. And that belief structures your reality.

Let's say you try to direct the most critical opinion you can onto the Proposition:

* "Palmer's Proposition is false and misleading. I don't believe it, nor should you."

You can add any other pejorative descriptive language you wish, but that statement sums up the opinion.

What happens? If you speak the truth and truly don't believe Palmer's Proposition, then you experience that it is not true. It must be wrong or perhaps Palmer is seriously delusional and speaking nonsense. Otherwise you are lying or just pretending to

respond to his assertion. But then if you speak the truth, your experience corresponds to your belief, and the statement still reflects your true experiential reality and that simply provides further evidence that Palmer's Proposition is true. Your experience still depends on what you believe.

Perhaps you are convinced that your beliefs are based on experience rather than the other way around. In that case it is possible to wonder whether you are using your experiences as evidence to prove that your beliefs are true. To test that, perhaps you wipe the slate clean and simply observe. You begin to have experiences, and then you begin to interpret those experiences. Your interpretations are definitions that you as an observer apply to your experiences to classify and understand them. If you do not apply any definitions and interpretations to experiences, what can you think or say about them? Perhaps you have allowed others to indoctrinate you with beliefs that you should interpret experience situation *A* in such and such a way.

In any case experience is undefined until you decide how to define and interpret it. Whether you make up the definition yourself or borrow one from someone else, it does not matter. Experience becomes what you believe it is only after you define it with a belief.

Thus, whatever you think of Palmer's Proposition, once you hear it and understand the words, you must accept it and begin to incorporate it into your life or else play a game and pretend to ignore it. In other words, you know it is true, but you lie and say it isn't, and that it is stupid. Or you just pretend to ignore it and go on with your life – which of course is OK if that is how you prefer things to go. Disbelief in Palmer's Proposition leads a person to be vulnerable to whatever events he happens to experience. Belief in Palmer's Proposition at least opens up the possibility of doing something about the events of one's life. Why would a person deliberately choose to make herself vulnerable to the vagaries of the environment and society by disconnecting her consciousness from reality?

Perhaps Palmer's Principle defines your experience as encountering a proposition that reveals something you don't want to face, so you criticize it, and what else can you do but pretend to ignore it?

Do you see what I mean? "This statement is false" knocks you out. You are left feeling blank, without an answer. It may even reduce your confidence in logical reasoning. Is it true or not? I don't know. Is it indeterminate....? It becomes gobbledygook. You begin to grow suspicious of logic, as happened to mathematicians who became aware of the logical problems with many self-referring statements. Maybe somebody's lying or playing a trick on you. Maybe it's just a language game. We have to tighten up our logical thinking.

Palmer's Proposition can wake you up. You may realize:

I guess my beliefs and experiences intimately correspond. They may even be varying degrees of the same thing. But I'm free to believe whatever I want about the statement

or anything else, and Harry's OK, and I'm OK, and so is everyone else, even though we may not agree on our beliefs and/or experiences. (Of course, I can believe that I am right and others are wrong, and even that those who are wrong should be punished or otherwise dealt with.) I can believe whatever I want, but by denying belief in the link between belief and experience, I put myself in a state of deliberate self-disempowerment and pretense, since that denial is the assertion of a belief -- coupled with a denial of responsibility for holding that belief.

Interestingly, every time I create or take on a belief, or make a judgment about a belief or an experience, I stick another belief onto my belief system, which is a decision for which I must take full responsibility. If I do not take responsibility, I will continue blaming problems on others. Eventually I may wonder how it is that others are always "wronger" than I am. Or if it is all about God, how do I know that God is so right and perfect? If my beliefs determine my experiences, I can experience whatever I prefer by simply deciding what to believe and it is up to me to be honest and make sure that my beliefs deliberately match my experiences. I can take it further and begin to deliberately create the reality that I prefer and judge the success of my creative powers by how well my experiential reality matches what I prefer it to be -- not only believe what it **ought** to be, but believe with certainty that it **is** and adjust my life until the experiences match how I believe it really is.

If my beliefs do not determine my experiences, then I am subject to experiences determined by other people and other environmental forces beyond my control. However, the decision for it to be that way (which ever way I believe it to be) is my own responsibility based on holding the belief to that effect. There is no passing the buck.

Palmer's paradoxical sounding conundrum leads to a constructive result, not a meltdown of the whole system. The idea that there are various types of paradoxes is something that needs to be studied more. What we have just discussed may be just the tip of the iceberg and something highly relevant for the ongoing research into modern chaos theory, fractals, and other iterative systems. Hmm. Logical fractals. Can beliefs initiate experiences at various different scales of reality?

The alternative to the notion that beliefs determine experience leads to an interesting exposure of pretense. People say, "That's preposterous. My beliefs are generated by my experiences. I get my beliefs from God, Jesus, Moses, Mohamed, Buddha, Holy Scripture, my parents, my teachers, my guru, my boss, my friends, my spouse, my government, my enemies, and, by knocking around in the world, I get them from Nature."

All of this is missing the point, since people who react this way have first chosen to believe that they got their beliefs from sources other than themselves. They are playing the game where you disallow yourself as source and decide to let yourself be indoctrinated by the people and environment with which you have chosen to associate.

* "I chose to accept the beliefs people offered me so I could experience what it is like to be indoctrinated." Well, feel what it feels like to be indoctrinated.

For a serious game player such a meta-belief may become what Palmer calls "transparent". A transparent belief acts like a glass wall. It limits you, but you don't recognize its existence because you look right through it. It's either too obvious, or you forgot about it, or you're just used to it, or some other excuse.

Every once in a while you bang your head against it. But that's OK. It's just a variation of the head-banging game. Logic is a game with words and rules -- a game of beliefs. People who play games soon learn that the rules are set up arbitrarily for enjoying the game. Scientists gradually learn the same thing. Mathematicians tend to catch on sooner and learn to try on different rules just to experience how they work, what they lead to, and what that all feels like. Logic, especially of the deductive variety, does not necessarily have anything to do with the physical world.

1. All students at this school are smart.
2. John is a student at this school.
3. Conclusion: John is smart.

This deduction may or may not be true, but it is logically valid. It is important to ascertain the truth value of any deductive statements before incorporating them into your belief system and acting on them. The conclusion is only as true as the truth value of the premises. We can say that truth is based in experience. In science, such arguments depend on accuracy of data and the truthfulness of the premises that lead to the conclusion. Data that do not fit the premises invalidate the truth of any hypothesis despite its logical validity on paper. Unfortunately, it is impossible to prove any hypothesis by empirical data. One may only strengthen an argument by finding data that tend to support the premises of a logical argument. Also, the mathematical apparatus of the model chosen may or may not be the most suitable one for supporting the hypothesis.

In other words, when logic links to experiences in the world, that is when you may bang your head. From Harry Palmer's viewpoint, all systems are sets of beliefs, even non-systems (self-contradictory, anarchic, chaotic, and so on). Some just happen to be empowering, and others are disempowering. But empowerment and disempowerment are only significant if you are into power games. Believing beliefs is more like a kind of game. It can be sane or insane. Insane games are games that self destruct and benefit few or none at all. Perhaps all games eventually self destruct in the sense of coming to an end -- win, lose, or draw. There is no justification for or against beliefs and games.

Exercise: Do #22, "Belief and Indoctrination" in **ReSurfacing**. Then find a partner to coach you in #23, "Transparent Beliefs".

What is the point of holding a belief? There is no value in it unless it results in some form of experience, even if it is only a mental experience. We end up with an extremely simple and general cycle of reality that consists of four phases that loop around over and over: asserting or re-asserting a selected **belief**, allowing that belief to **interact with the**

world as it is (thereby disturbing the balance of reality if the belief differs in any way from what already is), experiencing the **reaction** of the assertion of belief that rebounds from what is (the current set of beliefs and experiences), and **return to balance** in the state of how things are integrated with the totality of beliefs and experiences.

Four Phases in the Cycle of Reality

1. **Assert (or reassert) a belief by means of will.**
2. **Possibly disturb the balance of reality by the belief interacting with what is.**
3. **Loss of will in the reaction (or not) of what is to the asserted belief.**
4. **Return to balance as the asserted belief is integrated, or resist and return to stage one for another loop around the arena of life.**

Transparent beliefs are beliefs that an observer-participant has asserted or frequently reasserts without conscious deliberate intentionality. Or they may be forgotten, habituated, suppressed, but above all are not noticed, even in the face of strong experiential evidence. They may or may not disturb reality, result in loss of willpower, or generate resistance in a person. The main property of a transparent belief is that it exists but is transparent to the observer-participant and thus remains unperceived and uninspected.

The goal of theoretical physics is to uncover all the transparent beliefs we have about the nature of our physical world. The Holy Grail for many physicists is to come up with a simple and lucid explanation for everything, a unified theory, an ultimate formula for how physical reality works. But as Palmer points out, the importance of anything is "assigned by the observers and participants." (**ReSurfacing**, p. 64) In another little book by Palmer, **Living Deliberately**, (which you can download free from the Star's Edge web site www.AvatarEPC.com) on pp. 89-92 Palmer outlines a little set of "axioms" with an essay entitled "Viewpoint and the Nature of Being." At the end he laconically notes that,

* "The structure and mechanics of the physical universe may be extrapolated from these ideas."

These four brief pages on "Viewpoint . . ." by Palmer are definitely worth many careful reads, especially by anyone interested in physics. What Palmer does in those pages is give a simple list of basic definitions with statements of how they relate to each other. These terms include:

* create, define, experience, believe, universe, reality, awareness, consciousness, impression, creation, viewpoint, self, identity, and limit or boundary.

Interestingly, Palmer elects to identify reality with order and unreality with disorder, but I'm sure he would agree that the observer assigns the relative importance to such things.

Imagine the universe dying a heat death. There is just an expanding gas of random particles slowly cooling. Interactions drop toward zero. Entropy increases. Then the

quantum jiggles drop toward zero. Time has stopped long ago. Suddenly we become aware that the whole collection of "dead" particles has spontaneously phase locked just by virtue of being together in the same space, like pendulum clocks on a wall. Space/time is relative, and we have no frame to provide a reference point, so the whole shebang may as well be a tiny mote inside the Planck diameter. Uh Oh! We just G nab Gibbed!! Wham! Before you realize it, the Big Bang occurs, and we are off on another universe cycle, surfing again down from a Poincaré Peak into the Sea of Entropy.

Boltzmann fans laugh at the idea of all the 10^{23} molecules of air in a room suddenly jumping into a corner. But a Poincaré Recursion even in the vast phase space of the whole universe is not as unlikely as it might seem once we start playing with powerful tools such as non-linearity, observer-assigned frames, and the like. Even though it seems from the current popular viewpoint that the Big Bang after-flash (the epoch of recombination when excited electrons dropped into orbits around excited protons to form hydrogen and emitted a flash of photons that decoupled as radiation in space) observed by Penzias and Wilson (and estimated to have occurred around 378,000 years after the Big Bang) is currently cooled to a cosmic microwave background with a temperature of around 3 Kelvin, the "hot" one is going on everywhere all the time in the virtual space/time of the vacuum state. We just miss it flashing by because that's not where our attention is tuned. We see the universe already spread out, expanded, still expanding, and cooling. (See **Wikipedia**, "Cosmic Microwave Background", and "Recombination (Cosmology)").

If you are following the gist of my discussion, then you are there in your imagination, and to some extent that is your experience. It has become your reality to the extent that you believe it. This is one way that realities can tunnel from one person's consciousness to another's as a bit of subtle indoctrination. Language is a type of phase locking between communicators. That is why I delved into the nature of language and its design features in the first chapter of Observer Physics.

Humans seem to have the most advanced language system that we know of, but there may be organisms in our universe with more advanced design features. These could be known or unknown organisms. The non-local or global phenomena we often see with chaotic and fractal systems may indicate higher order communication features. The same is true of Bohm's hypothesized "implicate order", a kind of transcendental ESP by which quantum particles can navigate in the field of all possibilities letting entangled or otherwise correlated particles know what is happening.